

# A System Identification Technique Based on the Random Decrement Signatures Part I: Theory and Simulation

*Nabih E. Bedewi*  
*Jackson C. S. Yang*

Identification of the system parameters of a randomly excited structure may be treated using a variety of statistical techniques. Of all these techniques, the Random Decrement is unique in that it provides the homogeneous component of the system response. Using this quality, a system identification technique was developed based on a least-squares fit of the signatures to estimate the mass, damping, and stiffness matrices of a linear randomly excited system. In this part of the paper the mathematics of the technique is presented in addition to the results of computer simulations conducted to demonstrate the prediction of the response of the system and the random forcing function initially introduced to excite the system. Part II of the paper presents the results of an experiment conducted on an offshore platform scale model to verify the validity of the technique and to demonstrate its application in damage detection.

## INTRODUCTION

In general, all system identification techniques begin by assuming a form for the equations describing the system, then attempt to identify the unknown parameters in that assumed system through prior knowledge of the actual response, and sometimes the input as well. For linear systems, the identification process could be conducted in two different ways depending on the available information. To describe the two methods, consider a multidegree-of-freedom system having the following set of differential equations:

$$[M] \ddot{\mathbf{X}} + [K] \mathbf{X} = \mathbf{F} \quad (1)$$

where  $\mathbf{X}$  is the response vector and  $\mathbf{F}$  is the input loading vector. The response vector may further be viewed as the sum of the homogeneous solution vector  $\mathbf{X}_h$  and the particular solution vector  $\mathbf{X}_p$ , i.e.

$$\mathbf{X} = \mathbf{X}_h + \mathbf{X}_p \quad (2)$$

The first way of identifying the system in Equation (1) is to measure the input into the system,  $\mathbf{F}$ , and the response of the system,  $\mathbf{X}$ . Then through the use of a curve fitting technique, matrices  $[\mathbf{M}]$  and  $[\mathbf{K}]$  may be identified [14,15]. The disadvantage of this method is that the input must either be of a type that could be measured, or it must be deliberately introduced into the system. Furthermore, if the system has  $N$  degrees-of-freedom and only  $M$  locations are monitored, where  $M < N$ , these monitored locations must be selected specifically to include all external loads into the system. Therefore, this method is impractical in applications where the system is naturally excited, such as offshore structures impacted by wave motion, and flight vehicles excited by turbulent air flow.

The other approach for identifying the system parameters is through the use of the system response only [2,7,8,9,12]. This leads to the identification of the eigenvalues and eigenvectors of Equation (1) as opposed to the mass and stiffness matrices.

Substituting Equation (2) into Equation (1) yields

$$[\mathbf{M}] (\ddot{\mathbf{X}}_h + \ddot{\mathbf{X}}_p) + [\mathbf{K}] (\mathbf{X}_h + \mathbf{X}_p) = \mathbf{F}$$

which may be separated into two independent equations, namely

$$[\mathbf{M}] \ddot{\mathbf{X}}_p + [\mathbf{K}] \mathbf{X}_p = \mathbf{F} \quad (3)$$

$$[\mathbf{M}] \ddot{\mathbf{X}}_h + [\mathbf{K}] \mathbf{X}_h = \{0\} \quad (4)$$

Equations (3) and (4) indicate that if  $\mathbf{F}$  is not known, the system parameters may not be identified unless the homogeneous and particular solutions are separated. In practice, however, if the input spectral density is relatively flat over the range of the system frequencies, the ratios of the responses  $x_i(t)$  at the different locations are taken in the frequency domain thus yielding the eigenvalues and the eigenvectors of Equation (4). If, on the other hand, the input spectral density has some mild fluctuations over the frequency range of the system, then taking the ratios of the responses could yield erroneous eigenvectors. In addition, if the system exhibits some damping, the modal damping ratios may not be identified correctly. This is mainly due to the fact that the frequency content of the particular component of the response may vary considerably at different locations in the system.

These problems may be overcome by employing the Random Decrement (Randomdec) and cross-Random Decrement (cross-Randomdec) techniques [3,4,5,6,10,11,13]. Given the response vector  $\mathbf{X}$ , the Randomdec and corresponding cross-Randomdec signatures are calculated. Based on the results obtained in reference [1], the signatures should be interpreted as the homogeneous components of the response, namely  $\mathbf{X}_h$ . With this being the case, the eigenvalues and eigenvectors of Equation (4) may be estimated accurately. The biggest advantage of this method is that the response need not be measured at the points where external loads are applied. Furthermore, modal damping may be evaluated accurately from the Randomdec signatures either in the time domain using the logarithmic decrement or in the frequency domain using the half power point (curve fitting could generally be used for close modes in both domains).

Although modal parameters might be sufficient for many applications, it is sometimes desirable, if not necessary to have information reflecting the actual mass,

damping, and stiffness of the system. This could be the case in systems where damping is not proportional, or when the model is required for damage detection in which elements in the original matrices pertain to actual locations in the system. A technique is therefore proposed to estimate the  $[M]$ ,  $[C]$ ,  $[K]$  matrices of a linear system with the use of the Random Decrement technique.

### PROPOSED SYSTEM IDENTIFICATION TECHNIQUE

Consider the linear set of equations

$$[M] \ddot{\mathbf{X}} + [C] \dot{\mathbf{X}} + [K] \mathbf{X} = \mathbf{F} \quad (5)$$

where  $[M]$  and  $[K]$  are real symmetric matrices and  $[C]$  is a nonproportional, real, symmetric damping matrix. Introducing matrix  $H_{pij}$  and vector  $Z_{pj}$ , where

$$\begin{aligned} H_{1ij} &= [M] & Z_{1j} &= \ddot{\mathbf{X}} \\ H_{2ij} &= [C] & Z_{2j} &= \dot{\mathbf{X}} \\ H_{3ij} &= [K] & Z_{3j} &= \mathbf{X} \end{aligned}$$

Equations (5) may be rewritten in the form

$$\sum_{j=1}^M \sum_{p=1}^3 H_{pij} Z_{pj} = F_i \quad i = 1, 2, \dots, M \quad (6)$$

where  $M$  is the number of degrees-of-freedom in the system, and  $F_i$  symbolizes element  $i$  of vector  $\mathbf{F}$ .

If  $Z_{pj}$  and  $F_i$  are composed of  $N$  discrete points in time, there should exist one equation similar to Equation (6) for every point  $k$ , where  $k = 1, 2, \dots, N$ . Therefore, for one time step  $k$

$$\sum_{j=1}^M \sum_{p=1}^3 H_{pij} Z_{pjk} = F_{ik} \quad \begin{aligned} i &= 1, 2, \dots, M \\ k &= 1, 2, \dots, N \end{aligned} \quad (7)$$

To identify the three matrices  $H_{pij}$ , a least squares scheme will be employed to obtain the best estimate for  $H_{pij}$  through minimizing the difference between the left and right sides of Equation (7). Therefore, defining an error index  $e_{ik}$  for each equation  $i$  at every time step  $k$ , Equation (7) may be rearranged as follows :

$$F_{ik} - \sum_{j=1}^M \sum_{p=1}^3 \tilde{H}_{pij} Z_{pjk} = e_{ik} \quad \begin{matrix} i = 1, 2, \dots, M \\ k = 1, 2, \dots, N \end{matrix} \quad (8)$$

where  $\tilde{H}_{pij}$  is the best estimate for  $H_{pij}$ .

Adding the sum of the squares of Equation (8), the total error  $E$  in the system may be defined as

$$\sum_{i=1}^M \sum_{k=1}^N \left[ F_{ik} - \sum_{j=1}^M \sum_{p=1}^3 \tilde{H}_{pij} Z_{pjk} \right]^2 = \sum_{i=1}^M \sum_{k=1}^N e_{ik}^2 = E \quad (9)$$

To minimize the error with respect to  $\tilde{H}_{pij}$ , the slope of Equation (9) relative to all the unknown parameters must be set to zero, i.e.

$$\frac{\partial E}{\partial \tilde{H}_{pij}} = 0 \quad \begin{matrix} p = 1, 2, 3 \\ i = 1, 2, \dots, M \\ j = 1, 2, \dots, M \end{matrix}$$

Therefore, taking the partial derivative of  $E$  with respect to  $\tilde{H}_{pij}$  and noting that  $\tilde{H}_{pij} = \tilde{H}_{pji}$  (symmetry condition), the following two equations result

$$\frac{\partial E}{\partial \tilde{H}_{pij}} = -2 \sum_{k=1}^N Z_{pik} \left[ F_{ik} - \sum_{n=1}^M \sum_{q=1}^3 \tilde{H}_{qin} Z_{qnk} \right] = 0 \quad (10a)$$

for  $i = j$ , and

$$\begin{aligned} \frac{\partial E}{\partial \tilde{H}_{pij}} = & -2 \sum_{k=1}^N \left\{ Z_{pjk} \left[ F_{ik} - \sum_{n=1}^M \sum_{q=1}^3 \tilde{H}_{qin} Z_{qnk} \right] \right. \\ & \left. + Z_{pik} \left[ F_{jk} - \sum_{n=1}^M \sum_{q=1}^3 \tilde{H}_{qin} Z_{qnk} \right] \right\} = 0 \end{aligned} \quad (10b)$$

for  $i \neq j$ .

Equations (10a) and (10b) form a set of  $M(M+1)3/2$  linear simultaneous equations which may be solved either in closed form or by iteration. To use the latter, the partial derivatives must be taken of Equations (10a) and (10b) with respect to all the unknown parameters resulting in a constant Jacobian matrix, namely

$$\frac{\partial^2 E}{\partial \tilde{H}_{pij} \partial \tilde{H}_{ghl}} = 2 \sum_{k=1}^N Z_{pjk} Z_{glk}$$

for  $i = j$  , and

$$\frac{\partial^2 E}{\partial \tilde{H}_{pij} \partial \tilde{H}_{ghl}} = 2 \sum_{k=1}^N [ Z_{pjk} Z_{glk} + Z_{pik} Z_{ghk} ]$$

for  $i \neq j$  .

To obtain a solution in closed form, Equations (10a) and (10b) should be rewritten in the form

$$\sum_{k=1}^N \sum_{n=1}^M \sum_{q=1}^3 \tilde{H}_{qin} Z_{pik} Z_{qnk} = \sum_{k=1}^N Z_{pik} F_{ik} \quad (11a)$$

for  $i = j$  , and

$$\sum_{k=1}^N \sum_{n=1}^M \sum_{q=1}^3 [\tilde{H}_{qin} Z_{pjk} Z_{qnk} + \tilde{H}_{qjn} Z_{pik} Z_{qnk}] = \sum_{k=1}^N [ Z_{pjk} F_{ik} + Z_{pik} F_{jk} ] \quad (11b)$$

for  $i \neq j$ , where the right side of Equations (11a) and (11b) contains the constant terms while the left side is a constant coefficient matrix premultiplied by a vector composed of the  $M(M+1)3/2$  unknown system parameters. This may be represented as follows:

$$[G] \mathbf{h} = \tilde{\mathbf{F}} \quad (12)$$

These equations can now be solved using any conventional linear equation solver.

It is apparent by inspection of Equation (12) that when the Random Decrement technique is applied,  $\tilde{\mathbf{F}}$  reduces to zero , thus resulting in

$$[G] \mathbf{h} = \{0\}$$

or, in expanded form

$$\begin{bmatrix}
 g_{1,1} & g_{1,2} & \dots & g_{1,\beta-1} & g_{1,\beta} \\
 g_{2,1} & \dots & \dots & \dots & g_{2,\beta} \\
 \cdot & \cdot & & & \cdot \\
 \cdot & & \cdot & & \cdot \\
 \cdot & & & \cdot & \cdot \\
 \cdot & & & & \cdot \\
 \cdot & & & & \cdot \\
 g_{\beta-1,1} & \dots & \dots & \dots & g_{\beta-1,\beta} \\
 g_{\beta,1} & g_{\beta,2} & \dots & g_{\beta,\beta-1} & g_{\beta,\beta}
 \end{bmatrix}
 \begin{bmatrix}
 h_1 \\
 h_2 \\
 \cdot \\
 \cdot \\
 \cdot \\
 \cdot \\
 \cdot \\
 h_{\beta-1} \\
 h_{\beta}
 \end{bmatrix}
 =
 \begin{bmatrix}
 0 \\
 0 \\
 \cdot \\
 \cdot \\
 \cdot \\
 \cdot \\
 \cdot \\
 0 \\
 0
 \end{bmatrix}
 \quad (13)$$

where  $\beta = M(M + 1)3/2$ . Equations (13) are in homogeneous form and therefore do not possess a unique solution for  $\mathbf{h}$ .

On the other hand, if the rank of  $[G]$  is  $\beta - 1$ , and if one of the unknown parameters in  $\mathbf{h}$  were indeed known, then a unique solution would exist. This may be proved by rearranging Equation (13) as follows :

$$\begin{bmatrix}
 g_{1,1} & g_{1,2} & \dots & g_{1,\beta-1} \\
 g_{2,1} & \dots & \dots & g_{2,\beta-1} \\
 \cdot & \cdot & & \cdot \\
 \cdot & & \cdot & \cdot \\
 \cdot & & & \cdot \\
 \cdot & & & \cdot \\
 \cdot & & & \cdot \\
 g_{\beta-1,1} & \dots & \dots & g_{\beta-1,\beta-1}
 \end{bmatrix}
 \begin{bmatrix}
 h_1 \\
 h_2 \\
 \cdot \\
 \cdot \\
 \cdot \\
 \cdot \\
 \cdot \\
 h_{\beta-1}
 \end{bmatrix}
 =
 \begin{bmatrix}
 -g_{1,\beta} h_{\beta} \\
 -g_{2,\beta} h_{\beta} \\
 \cdot \\
 \cdot \\
 \cdot \\
 \cdot \\
 \cdot \\
 -g_{\beta-1,\beta} h_{\beta}
 \end{bmatrix}
 \quad (14)$$

where  $h_{\beta}$  was assumed to be the known parameter for convenience. Equation (14) may be set up for any known  $h_i$  by simply eliminating the  $i^{\text{th}}$  row of  $[G]$  (since the partial derivative with respect to a constant is zero) then moving the  $i^{\text{th}}$  column to the right side and multiplying it by the scalar  $-h_i$ . The only condition for selecting  $h_i$  is that the  $i^{\text{th}}$  row must increase the rank of the system. With the aforementioned steps adhered to, Equation (14) will definitely have a unique solution.

If the rank of matrix  $[G]$  is less than  $\beta - 1$ , the solution of Equation (13) becomes more difficult to obtain. The linear dependency of the equations forming matrix  $[G]$  will require a greater effort to be eliminated, and, more parameters of vector  $\mathbf{h}$  will have to be assumed. Therefore, if the rank is  $\beta - 2$ , two equations from Equations (13) must be removed and two parameters of vector  $\mathbf{h}$  must be assumed. Calculating two or more parameters in many physical systems is extremely difficult, if not impossible. The examples used in the remainder of this paper are all of systems with a rank of  $\beta - 1$ .

The question to be raised then is how can the value of one of the parameters be known?

Since  $\mathbf{h}$  is composed of elements from the mass, damping, and stiffness matrices, it is physically immeasurable. Therefore, the best alternative would be to obtain a good estimate. The requirement imposed on the accuracy of the estimate depends greatly on the application for which the model is needed. If the model is to be used for damage detection, then knowing the absolute values of the  $[M]$ ,  $[C]$ ,  $[K]$  matrices is not essential, but rather, the ratio of the values of the elements at different instances in time. Therefore, assuming the order of magnitude of one of the elements in  $\mathbf{h}$  should suffice as long as the same value is used every time. On the other hand, if the model is needed for simulation purposes, where one wishes to study the effect of different loading conditions on the response of the system, the absolute values are needed. In this case, the estimate for  $h_1$  may be obtained from either a finite element model, or by performing a simple static test on the system (if physically possible). The latter is carried out by applying a known static load at a point in the system and measuring the deflection at the same point or at any other point. These two values may then be used to scale the entire  $\mathbf{h}$  vector ( $\mathbf{h}$  must be already calculated by assuming the order of magnitude of  $h_1$ ). The scaling procedure may be described by noting the static component of Equation (5), namely

$$[K] \mathbf{X} = \mathbf{F} \quad (15)$$

If  $[K]$  is separated into a scalar  $\epsilon$  multiplying a matrix  $[\tilde{K}]$ , where  $[\tilde{K}]$  is the estimated stiffness matrix using the proposed method, then Equation (15) becomes

$$\epsilon [\tilde{K}] \mathbf{X} = \mathbf{F} \quad (16)$$

Since  $\mathbf{F}$  and  $[\tilde{K}]$  are known ( $\mathbf{F}$  is the force applied during the static test) the system of equations

$$[\tilde{K}] \tilde{\mathbf{X}} = \mathbf{F}$$

may be solved to give  $\tilde{\mathbf{X}}$ , where

$$\tilde{\mathbf{X}} = \epsilon \mathbf{X}$$

But, the response  $X_i$  at point  $i$  in the system was measured, therefore  $\epsilon$  may be obtained by simply taking the ratio  $\tilde{X}_i/X_i$ . The value of  $\epsilon$  may then be multiplied by the estimated mass, damping, and stiffness matrices  $[\tilde{M}]$ ,  $[\tilde{C}]$ , and  $[\tilde{K}]$  respectively to obtain  $[M]$ ,  $[C]$ , and  $[K]$ .

### TESTS OF PROPOSED TECHNIQUE BY COMPUTER SIMULATION

To demonstrate the procedure for applying the proposed system identification technique, and to test its accuracy, a six degree-of-freedom model of a cantilever beam is used. Figure 1 shows the beam with the locations of the six points where the response is monitored. Using a finite element model (Reference [15]), the system matrices in Equation (5) were obtained.

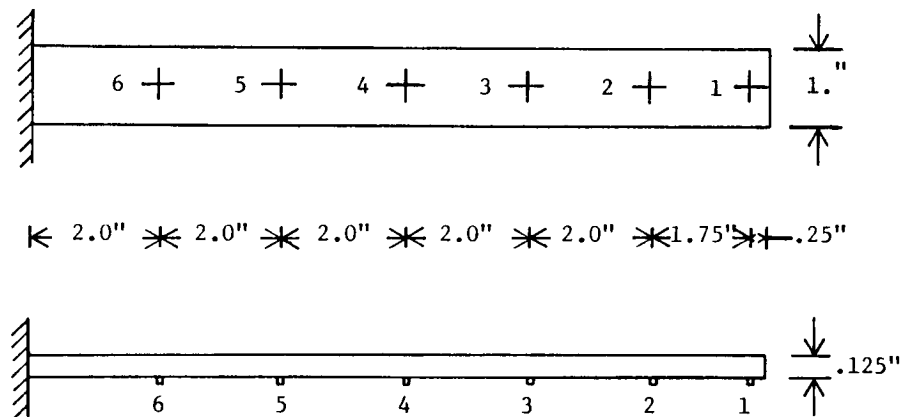


Figure 1 - Locations of monitored points on the cantilever beam

To distinguish between errors introduced by the system identification technique and the errors in the Randomdec signatures, two different scenarios were conducted. The technique was initially tested using exact free-decay response curves of the system and then tested using actual Randomdec signatures obtained from the random response of the system.

#### Case I: Exact Free-Decay Curves

A set of initial conditions was arbitrarily chosen for the six locations on the beam. Equation (5) was then solved numerically and the response vectors  $\mathbf{X}$ ,  $\mathbf{X}$ , and  $\mathbf{X}$  were recorded. The time step size was selected to insure that at least seven points were needed to construct one cycle of the highest frequency in the system. Solving for the eigen-values of Equation (5) the undamped natural frequencies of the system were found to be

$$f_1 = 25.908 \text{ Hz}$$

$$f_4 = 886.770 \text{ Hz}$$

$$f_2 = 159.251 \text{ Hz}$$

$$f_5 = 1445.039 \text{ Hz}$$

$$f_3 = 445.307 \text{ Hz}$$

$$f_6 = 2114.293 \text{ Hz}$$



thus resulting in a step size of  $\Delta T = 0.00007$  seconds. Furthermore, the number of time steps had to be selected to cover at least one cycle of the lowest frequency in the system. Therefore, 600 points were used.

One of the tests for the technique is its repeatability relative to different fixed parameters, i.e. as elements  $h_1$  through  $h_{63}$  are in turn fixed. Since diagonal terms are more reliable than off-diagonal terms in the model, this test was strictly confined to the diagonal elements.

Equation (14) was solved eighteen times, each time fixing one of the diagonal elements in the  $[M]$ ,  $[C]$ ,  $[K]$  matrices. The fixed value was always taken as the actual value of the element to avoid scaling the  $\mathbf{h}$  vector. After every evaluation of the  $\mathbf{h}$  vector, the errors occurring in the diagonal terms, relative to their actual values, were calculated. Calculations were made of the errors in the mass, damping, and stiffness matrices as each diagonal element in the mass matrix was fixed. Similar results were obtained as each diagonal element in the damping and stiffness matrices was fixed, respectively. In all the cases, the average error occurring at every point on the beam was evaluated.

Following a careful inspection of the errors, two conclusions were made, viz. fixing the stiffness matrix gives better estimates of vector  $\mathbf{h}$  than fixing the mass and damping matrices, and, due to the large variation in the error resulting from fixing different elements in the same matrix, the average value is probably a more consistent estimate. Therefore, based on these conclusions, and noting that fixing the stiffness matrix elements is the only case in which the average gives a better overall estimate than the individual estimates, it is further concluded that the best approach for estimating vector  $\mathbf{h}$  is to take the average of the  $\mathbf{h}$  vectors obtained by individually fixing the stiffness diagonal elements. In doing so, the  $\mathbf{h}$  vectors must be scaled independently before the averaging process is carried out. Using this procedure, the system matrices were identified. These matrices were not identical to the actual system matrices; their validity to represent the system was checked by comparing the response to the same input. Therefore, a random input vector  $\mathbf{F}$  with constant spectral density was simulated on the computer and used as input into Equation (5). This equation was solved using the actual and the estimated system matrices. Results were obtained for all the response points. Comparison of the actual and predicted responses at point 1 is shown in Figure 2. The responses of the two systems compare favourably.

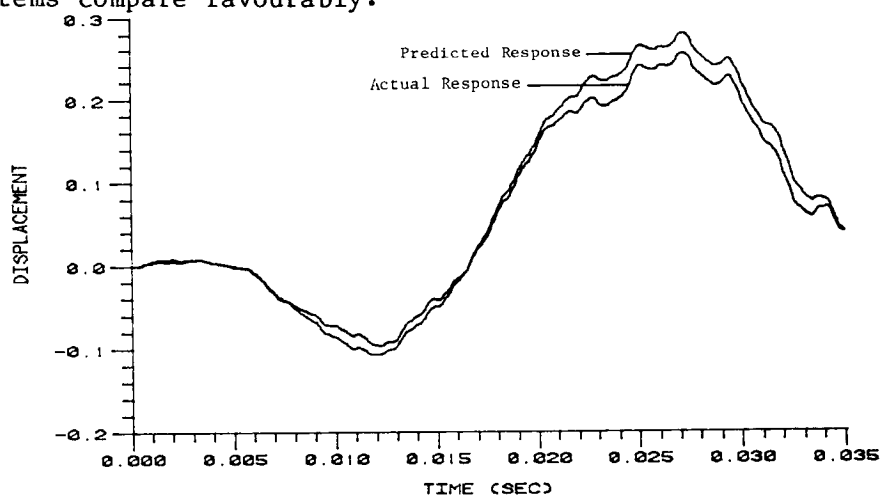


Figure 2 - Comparison of actual and predicted responses at point 1 (system identification from actual free-decay response)

### Case II: Free-Decay Curves From Randomdec Signatures

To further evaluate the overall accuracy of the system identification technique, the technique had to be tested by incorporating Randomdec and cross-Randomdec signatures as opposed to actual free-decay response curves.

Two purely uncorrelated stationary, Gaussian, random records were used as input forces at points 2 and 4 on the cantilever beam. Equation (5) was solved and the response vector  $\mathbf{X}$  recorded. Due to its high frequency content, station 6 was used as the triggering station with a trigger level of 0.0075. Five cross-Randomdec signatures (at stations 1 to 5), and one Randomdec signature (at station 6) were obtained for 600 lag points and 500 averaged segments. The cross-Randomdec signature for station 1 is shown in Figure 3. The first and second derivatives were then calculated for the six signatures using a finite difference scheme with an error on the order of  $\Delta T^2$ .

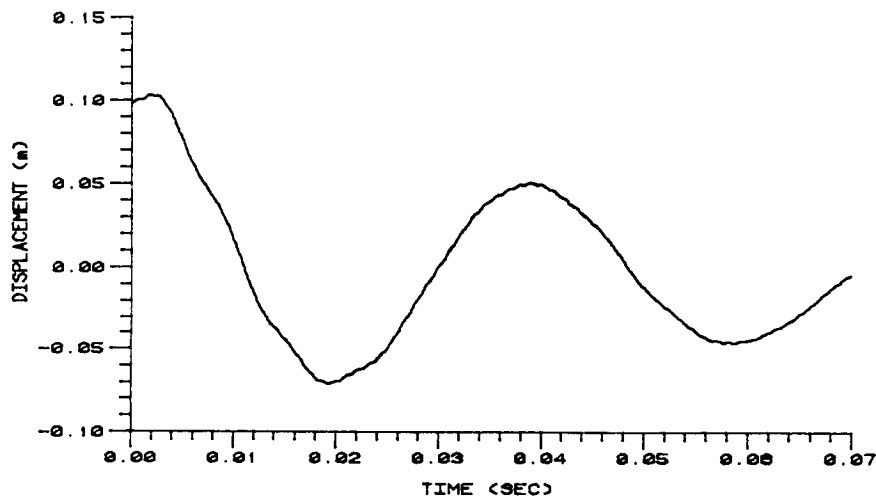


Figure 3 - Cross-Randomdec signature of time record at point 1

The procedure recommended in the previous section was used. Equation (14) was solved six times, each time fixing one of the diagonal elements in the stiffness matrix.  $K_{11}$  was fixed at 1000,  $K_{22}$  was fixed at 5000, and  $K_{33}$  through  $K_{66}$  were fixed at 10000. In each case, a load of 10 was applied at point 1 and, using the corresponding estimated stiffness matrix, the deflection at point 1 was calculated (since point 1 is the free end of the cantilever beam). The same procedure was followed using the actual stiffness matrix. The six  $\mathbf{h}$  vectors were then scaled following the procedure outlined in the previous section, averaged, and rearranged in matrix form.

Again these matrices were not identical to the actual system matrices. To test their validity as a simulation tool, the estimated matrices were substituted into Equation (5). The force vector used to obtain the random records, from which the signatures were evaluated, was used as the input. Comparison of the calculated response versus the actual system response was obtained for all the stations. Results at stations 1 and 2 are shown in Figures 4 and 5. Once more, the results compare favourably.

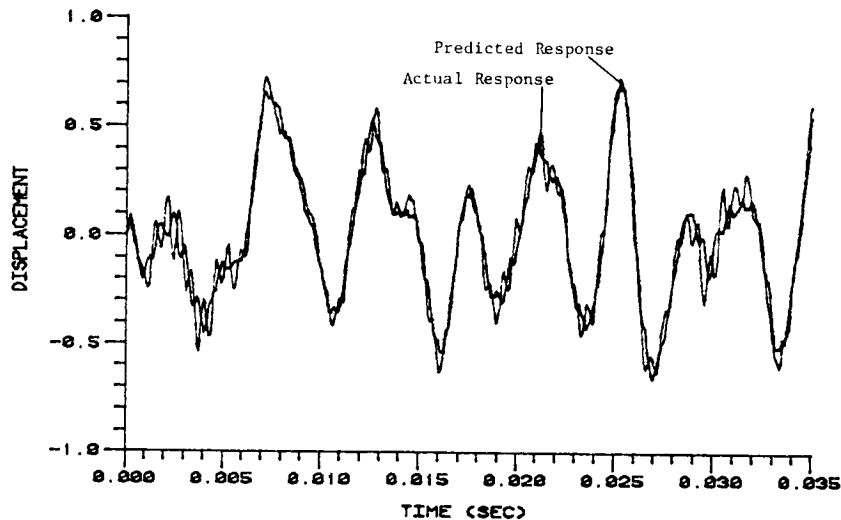


Figure 4 - Comparison of actual and predicted responses at point 1 (system identification from Randomdec signatures)

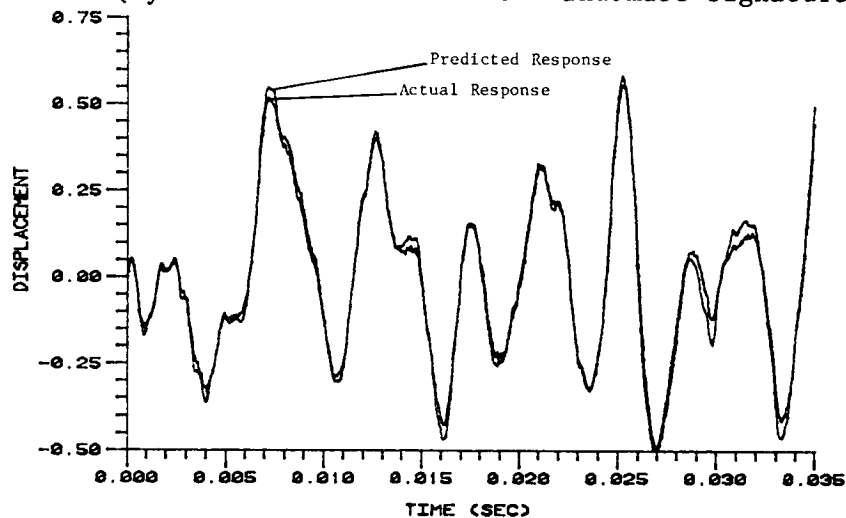


Figure 5 - Comparison of actual and predicted responses at point 2 (system identification from Randomdec signatures)

#### ESTIMATING THE INPUT INTO THE SYSTEM

An interesting application arises from the system identification technique by observing Equation (5). After estimating the mass, damping, and stiffness matrices, if the response vector  $\mathbf{X}$  and its derivatives are substituted back into Equation (5), the outcome should be the force vector  $\mathbf{F}$ .

Therefore, in a real application, the random response would be measured at several locations in the system. The signatures would then be obtained, their derivatives calculated, and Equation (14) used to estimate the  $[\mathbf{M}]$ ,  $[\mathbf{C}]$ ,  $[\mathbf{K}]$  matrices following the procedure outlined in the previous section. A direct substitution of the measured response and its derivatives into Equation (5) with the estimated matrices would result in a vector similar to the input vector. If the estimated mass, damping, and stiffness matrices were not scaled, the outcome of Equation (5) should be a scaled version of the input vector.

To demonstrate this approach, the example provided in the previous section was used. Results were obtained of the estimated input vector  $\mathbf{F}$  versus the actual input. Since the forces were originally applied at locations 2 and 4 on the beam, the forces at locations 1, 3, 5, and 6 should be zero. These Results indicate that at the points where the loads were applied, the estimated input functions formed good approximations (see Figures 6 and 7). As for the unloaded points, the technique predicted forcing functions with relatively small magnitudes in comparison to the loaded points (see Figure 8).

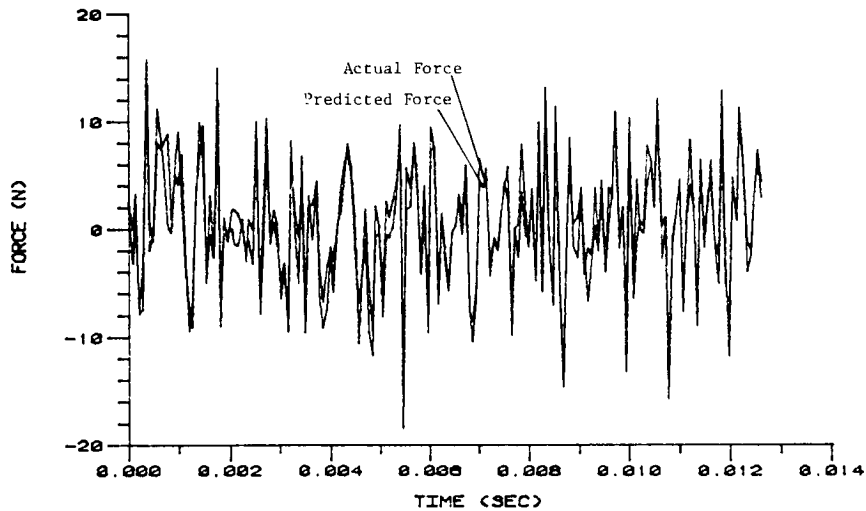


Figure 6 - Comparison of actual and predicted force records (record for location 2 - unfiltered response)

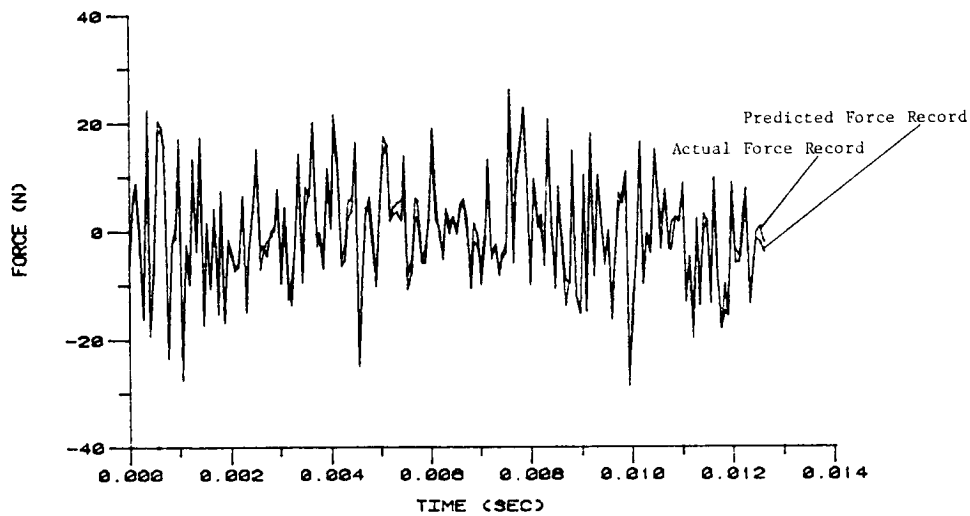


Figure 7 - Comparison of actual and predicted force records (record for location 4 - unfiltered response)

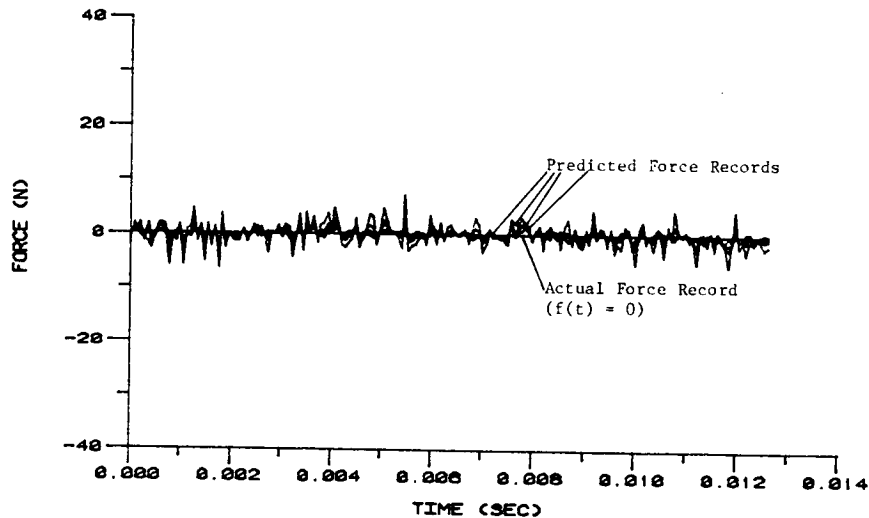


Figure 8 - Comparison of actual and predicted force records  
(records for locations 1,3,5, and 6 - unfiltered)

Care must be exercised when employing filters in this technique. If the signatures are obtained after filtering the response record, the response vector  $\mathbf{X}$  and its derivatives must also be filtered before substitution into Equation (5). If this procedure is not followed, the estimated force vector  $\mathbf{F}$  will also include the filtered modes of the system. This may be demonstrated using the same example. Studying the Fourier magnitude spectrums of the responses at points 1 and 6 and their derivatives, it was apparent that the lowest mode is quite dominant. Therefore, employing a high-pass filter at 80 Hz for the signatures, but not the response vector  $\mathbf{X}$ , the resultant estimated inputs at points 2 and 4 are shown in Figures 9 and 10. The low mode is quite apparent in the estimated input records. This problem could also occur when calculating signatures from velocity and acceleration records since they usually tend to include a larger density of the higher frequency modes.

In addition to the aforementioned effects, the frequency content of the response records is a major cause for the dissimilarity between the identified system matrices and the actual matrices. This may be explained by considering the six degrees-of-freedom cantilever beam. If the beam were excited by a random force with a band-limited frequency range, where, for the sake of example, this range included only the lowest three modes, then the identified matrices will possess information concerning these three modes only. This would mean that the identified  $[\mathbf{M}]$ ,  $[\mathbf{C}]$ , and  $[\mathbf{K}]$  matrices would definitely be different from the actual matrices. Therefore, subjecting the predicted system to the actual input vector should result in a response very similar to the measured system response, whereas a force rich in the higher frequencies would yield different results. Hence, using the proposed technique for damage detection requires the frequency content of the input force to always be the same.

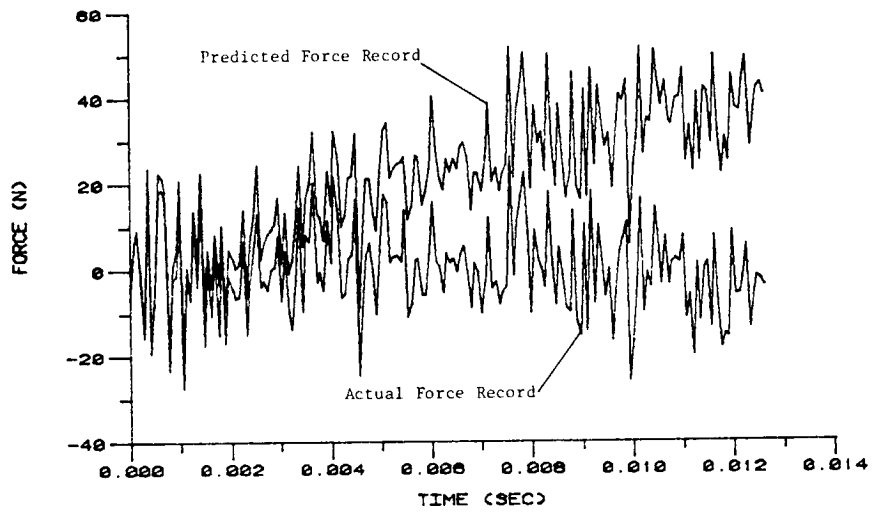


Figure 9 - Comparison of actual and predicted force records  
(record for location 2 - filtered response)

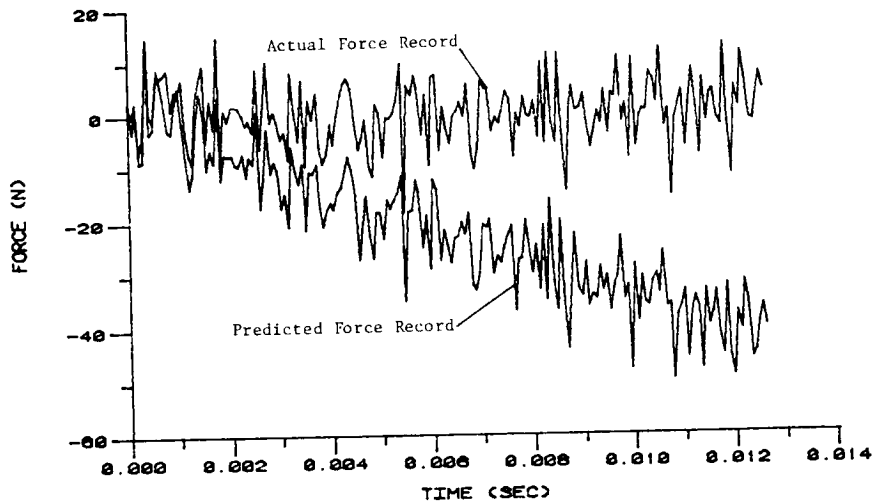


Figure 10 - Comparison of actual and predicted force records  
(record for location 4 - filtered record)

## CONCLUSIONS

A system identification technique was proposed based on a leastsquares fit of Randomdec and cross-Randomdec signatures to identify the mass, damping, and stiffness matrices of a linear multidegree-of-freedom system. Computer simulations carried out for a discretized finite element model of a cantilever beam proved the technique to be quite effective in predicting the response of the beam for a given frequency range of excitation. Furthermore, the proposed technique was demonstrated to be successful in predicting the random forcing function initially introduced to excite the system. The results of the simulation clearly indicated the importance of filtering the response of the beam and the effect it may have on the identified system.

## NOMENCLATURE

$[A]$	flexibility matrix
$[C]$	damping matrix of multiple D.O.F. system
$\tilde{[C]}$	unscaled identified damping matrix
$c_{ij}$	element $ij$ of damping matrix $[C]$
$F$	input loading vector
$f_i(t)$	forcing function applied at point $i$
$H_{pij}$	system matrix to be identified
$\tilde{H}_{pij}$	estimated system matrix
$h$	vector containing system parameters to be identified
$h_i$	element $i$ of vector $h$
$[K]$	stiffness matrix of multiple D.O.F. system
$\tilde{[K]}$	unscaled identified stiffness matrix
$k_{ij}$	element $ij$ of stiffness matrix $[K]$
$[M]$	mass matrix of multiple D.O.F. system
$\tilde{[M]}$	unscaled identified mass matrix
$m_{ij}$	element $ij$ of mass matrix $[M]$
$[R]$	matrix containing ratio of flexibility matrices
$r_{ij}$	element $ij$ of matrix $[R]$

$t$	time variable
$X$	response vector of multiple D.O.F. system
$X_h$	homogeneous response vector
$X_p$	particular response vector
$X_i$	system response at point $i$
$x(t), \dot{x}(t), \ddot{x}(t)$	position, velocity, and acceleration of variable $x$
$Z_{ij}$	measured system response
$\beta$	number of system parameters to be identified
$\epsilon$	scaling factor of identified system

#### REFERENCES

1. Bedewi, N.E., "The Mathematical Foundation of the Auto and Cross-Random Decrement Techniques and the Development of a System Identification Technique for the Detection of Structural Deterioration", Ph.D. Dissertation, University of Maryland, 1986.
2. Berman, A., "Parameter Identification Techniques for Vibrating Structures", The Shock and Vibration Digest, Vol. 11, No. 1, 1979, pp. 13-16.
3. Brignac, W.J., et al., "The Random Decrement Technique Applied to the YF-16 Flight Flutter Tests", AIAA/ASME/SAE 16th Structures Conf. Denver, Co., 1975.
4. Chang, C.S., "Study of Dynamic Characteristics of Aeroelastic Systems Utilizing Randomdec Signatures", NASA-CR 132563, 1975.
5. Cole, H.A., Jr., "Methods and Apparatus for Measuring the Damping Characteristics of a Structure", United States Patent No. 3,620,069, Nov. 16, 1971.
6. Cole, H.A., Jr., "Failure Detection of a Space Shuttle Wing Flutter Model by Random Decrement", NASA TMX-62,041, May 1971.
7. Crandall, S.H., Mark, W.D., Random Vibration in Mechanical Systems, Academic Press, New York, 1963.
8. Gersch, W., "Parameter Identification: Stochastic Process Techniques", The Shock and Vibration Digest, Vol. 7, No. 11, 1975, pp. 71-86.



9. Gersch, W. and Liu, R., "Time Series Methods for the Synthesis of Random Vibration Systems", ASME Journal of Appl. Mechanics, Vol. 43, 1976, pp. 159-165.
10. Hammond, C.E. and Dogget, R.V., Jr., "Determination of Subcritical Damping by Moving-Block/Randomdec Applications", Proc. of the NASA Symp. on Flutter Testing Techniques, Oct. 9-10, 1975.
11. Ibrahim, S.R. "Random Decrement Technique for Modal Identification of Structures", 18th Structures, Structural Dynamics, and Materials Conference, San Diego, CA., March 21-23, 1977.
12. Wang, Z. and Fang, T., "A Time-Domain Method for Identifying Modal Parameters", Journal of Appl. Mechanics, Vol. 53, March 1986, pp. 28-32.
13. Yang, J.C.S., Aggour, M.S., Dagalak, N. and Miller, F., "Damping of an Offshore Platform Model by Randomdec Method", Proc. Second ASCE/EMD Specialty Conference on Dynamic Response of Structures, Atlanta, Georgia, January 1981.
14. Yang, J.C.S., Tsai, T., Tsai, W.H., and Chen, R.Z. "Detection and Identification of Structural Damage from Dynamic Response Measurements", 4th Int'l Symp. on Offshore Mechanics and Arctic Engr., Dallas, TX, February, 1985.
15. Yang, J.C.S., Tsai, T., Pavlin, V., Chen, J., and Tsai, W.H., "Structural Damage Detection by the System Identification Technique", 55th Shock and Vibration Bulletin, June 1985.